



Whittaker
functions and
the alcove walk
model

Neelima Borade,
Matthew
Huynh, Henry
Twiss

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The Alcove
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An Introduction
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Whittaker functions and the alcove walk model

Neelima Borade, Matthew Huynh, Henry Twiss

University of Minnesota

July 2020



Outline

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Where Whittaker Functions Appear

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- Special functions over a split reductive group $G(F)$.



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- Special functions over a split reductive group $G(F)$.
- Historically, they referred to a solution to a confluent hypergeometric equation proposed by Whittaker.



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- Special functions over a split reductive group $G(F)$.
- Historically, they referred to a solution to a confluent hypergeometric equation proposed by Whittaker.
- Basic tool in aut. forms and the construction of L -functions.



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- Special functions over a split reductive group $G(F)$.
- Historically, they referred to a solution to a confluent hypergeometric equation proposed by Whittaker.
- Basic tool in aut. forms and the construction of L -functions.
- Arise as common eigenfunctions in physics.



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$G(F)$ is split reductive:



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$G(F)$ is split reductive: torus T ,



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$G(F)$ is split reductive: torus T , unipotent U ,



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$G(F)$ is split reductive: torus T , unipotent U , Borel $B = TU$,



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$G(F)$ is split reductive: torus T , unipotent U , Borel $B = TU$, maximal compact K .



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$G(F)$ is split reductive: torus T , unipotent U , Borel $B = TU$, maximal compact K . The Whittaker model is the space of functions W satisfying

$$W(ug) = \psi(u)W(g) \quad (\text{Whittaker relation}).$$



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$$W(ug) = \psi(u)W(g) \quad (\text{Whittaker relation}).$$

$G(F)$ acts by right translation.



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$G(F)$ acts by right translation.

Theorem 1.1 (Gelfand-Grave, Jacquet-Langlands, etc. al)

Given an irr. rep. (π, V) of $G(F)$ there is at most one isomorphic copy inside the Whittaker model under this action by right-translation.



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Theorem 1.1 (Gelfand-Grave, Jaquet-Langlands, etc. al)

Given an irr. rep. (π, V) of $G(F)$ there is at most one isomorphic copy inside the Whittaker model under this action by right-translation.

Upshot: We view (π, V) as functions on $G(F)$.



The Spherical Function

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Inside (π, V) there is a spherical vector ϕ_K fixed by the action of K .



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Inside (π, V) there is a spherical vector ϕ_K fixed by the action of K . Inside the Whittaker model ϕ_K is

$$W(t^\lambda) = \int_{U^-} v_K(ut^\lambda)\psi(u) du.$$



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Inside (π, V) there is a spherical vector ϕ_K fixed by the action of K . Inside the Whittaker model ϕ_K is

$$W(t^\lambda) = \int_{U^-} v_K(ut^\lambda)\psi(u) du.$$

This is the (spherical) Whittaker function.



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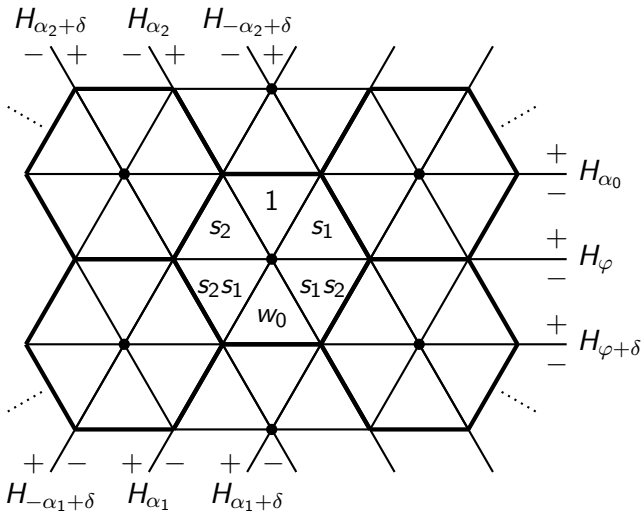
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We work in a Euclidean space $\mathfrak{h}_{\mathbb{R}}^*$.



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We work in a Euclidean space $\mathfrak{h}_{\mathbb{R}}^*$. Inner product (\cdot, \cdot) . Set

$$\langle \alpha, \beta \rangle = \frac{2(\alpha, \beta)}{(\alpha, \alpha)}.$$



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The alcoves are the triangles in the alcove diagram.



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The alcoves are the triangles in the alcove diagram.

$$W_{\text{aff}} \longleftrightarrow \{\text{alcoves}\}$$



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The alcoves are the triangles in the alcove diagram.

$$W_{\text{aff}} \longleftrightarrow \{\text{alcoves}\}$$

The affine Weyl group:

$$W_{\text{aff}} := \langle s_i \mid 0 \leq i \leq n \rangle$$



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$$W_{\text{aff}} \longleftrightarrow \{\text{alcoves}\}$$

The affine Weyl group:

$$W_{\text{aff}} := \langle s_i \mid 0 \leq i \leq n \rangle$$

where the s_i are reflections over certain hyperplanes.



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The affine Weyl group:

$$W_{\text{aff}} := \langle s_i \mid 0 \leq i \leq n \rangle$$

where the s_i are reflections over certain hyperplanes.

$$W := \langle s_i \mid 1 \leq i \leq n \rangle$$

is the finite Weyl group.



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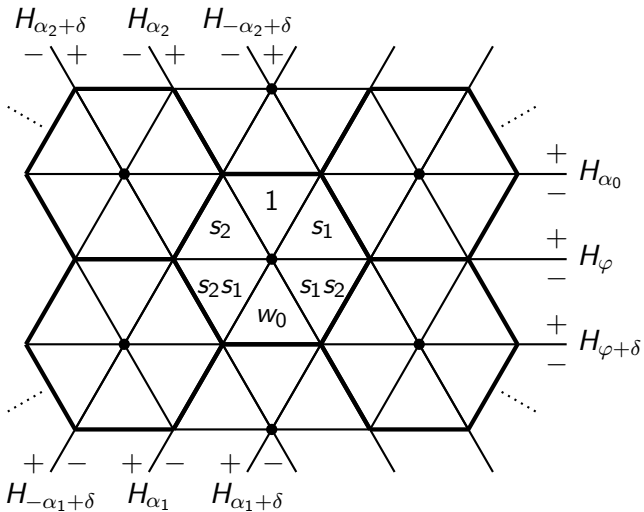
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The affine hyperplanes are

$$H_{\alpha_i + j\delta} := \{\beta \in \mathfrak{h}_{\mathbb{R}}^* \mid \langle \alpha_i, \beta \rangle = j\}.$$



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The affine reflection over $H_{\alpha_i + j\delta}$ is

$$s_{\alpha_i + j\delta}(\beta) := \beta - (\langle \alpha_i, \beta \rangle + j)\alpha_i.$$



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$\{\alpha_1, \dots, \alpha_n\}$ form a basis for a root system.



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$$W_{\text{aff}} = \langle s_i := s_{\alpha_i} \mid 0 \leq i \leq n \rangle.$$



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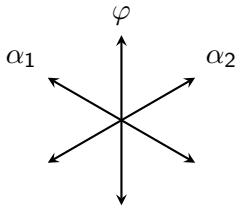
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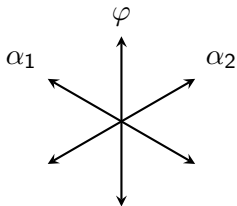
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$$W \cong S_3 \quad \text{and} \quad W_{\text{aff}} \cong \tilde{S}_3.$$



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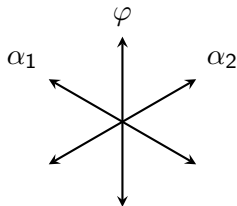
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$$W \cong S_3 \quad \text{and} \quad W_{\text{aff}} \cong \tilde{S}_3.$$

The highest root is

$$\varphi = \alpha_1 + \alpha_2.$$



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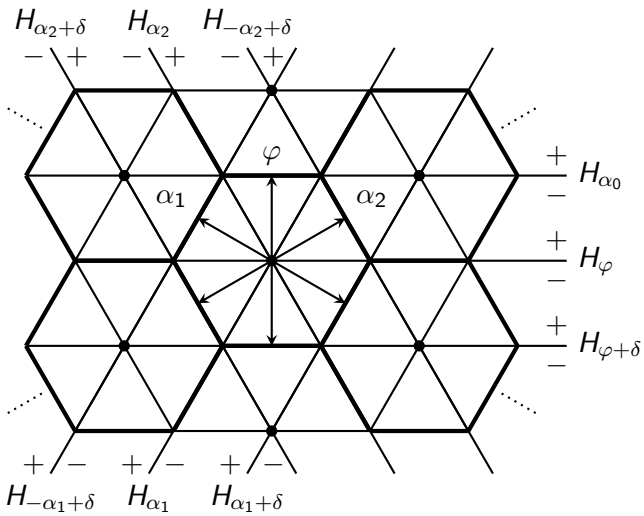
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The centers of hexagons are in bijective correspondence with Q^\vee :



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The centers of hexagons are in bijective correspondence with Q^\vee :

$$Q^\vee := \mathbb{Z}\alpha_1^\vee + \cdots + \mathbb{Z}\alpha_n^\vee$$



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$$Q^\vee := \mathbb{Z}\alpha_1^\vee + \cdots + \mathbb{Z}\alpha_n^\vee$$

where

$$\alpha_i^\vee := \frac{2\alpha_i}{(\alpha_i, \alpha_i)}.$$



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$$Q^\vee := \mathbb{Z}\alpha_1^\vee + \cdots + \mathbb{Z}\alpha_n^\vee$$

where

$$\alpha_i^\vee := \frac{2\alpha_i}{(\alpha_i, \alpha_i)}.$$

Also,

$$W_{\text{aff}} \cong W \rtimes Q^\vee$$

under translation by Q^\vee .



The Case for $5l_3$

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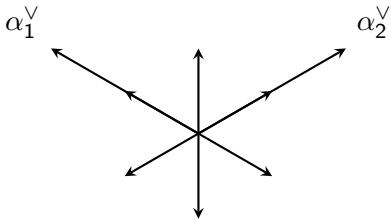
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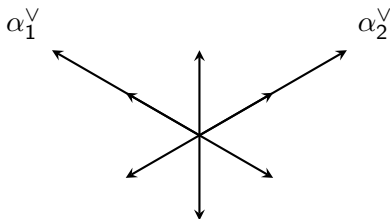
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$$Q^\vee = \mathbb{Z}\alpha_1^\vee + \mathbb{Z}\alpha_2^\vee \quad \text{and} \quad \tilde{S}_3 \cong S_3 \rtimes Q^\vee.$$



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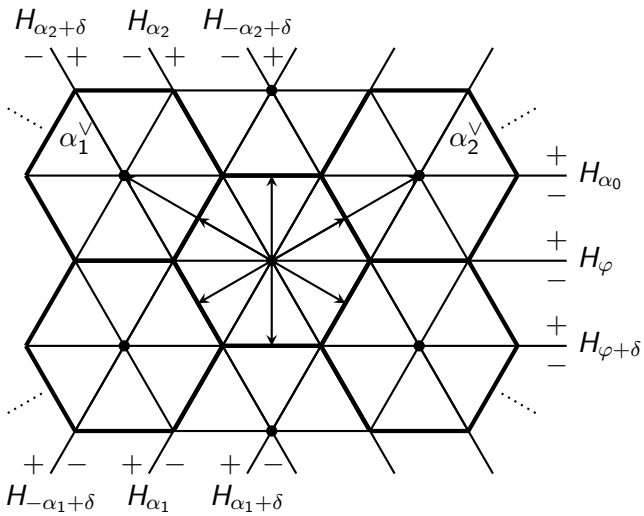
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Hyperplane Orientation

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Hyperplane Orientation

- 1 lies on the positive side of the H_{α_i}

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Hyperplane Orientation

- 1 lies on the positive side of the H_{α_i}
- $H_{\alpha_i+j\delta}$ and H_{α_i} have parallel orientations.

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- 1 lies on the positive side of the H_{α_i}
- $H_{\alpha_i+j\delta}$ and H_{α_i} have parallel orientations.

These facts dictate most of the combinatorics about the walk.



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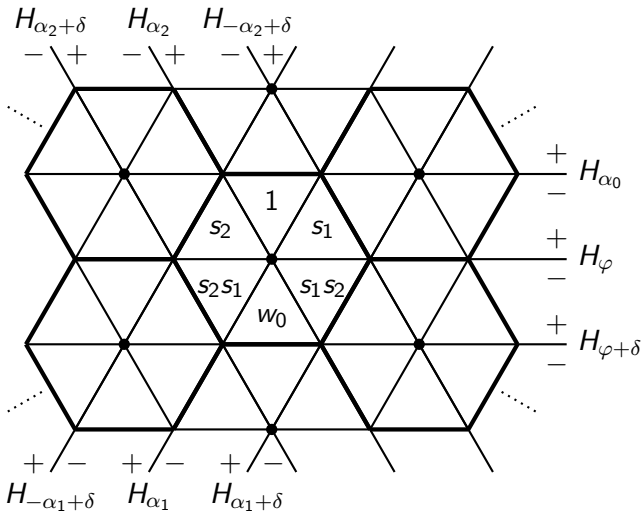
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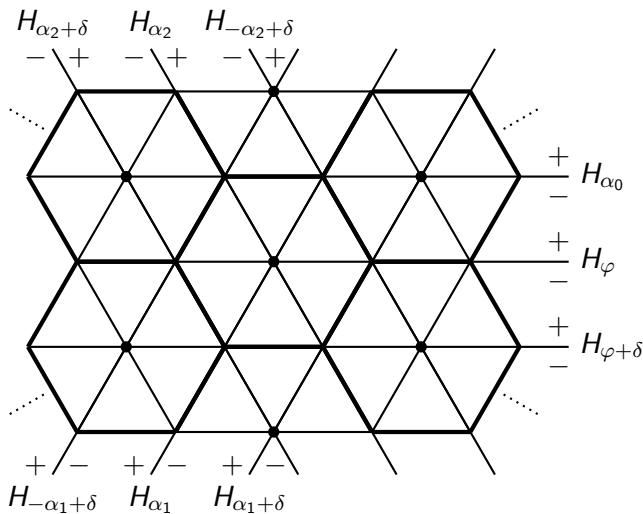
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The walk for $w = s_1 s_2 s_0 s_1$ is





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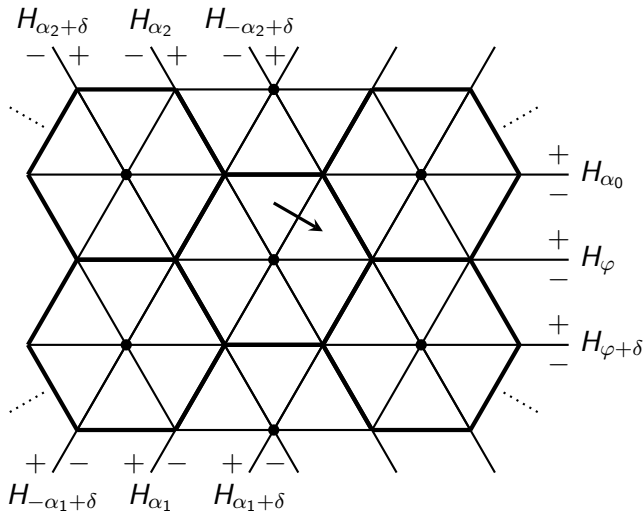
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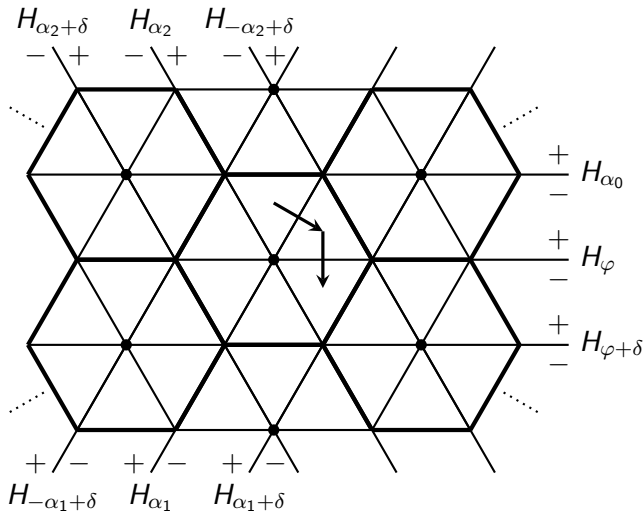
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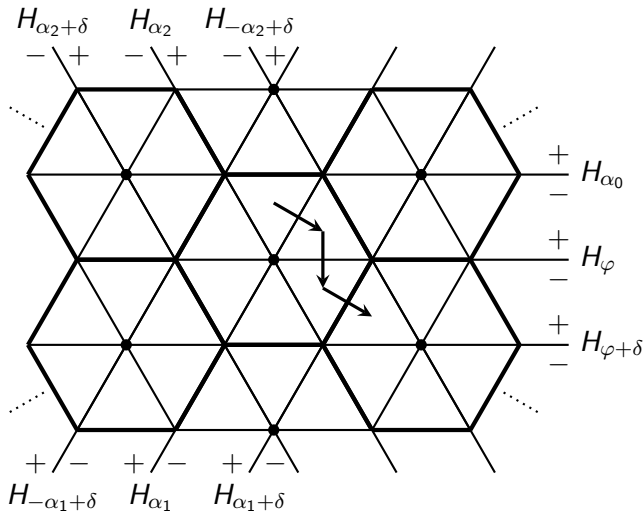
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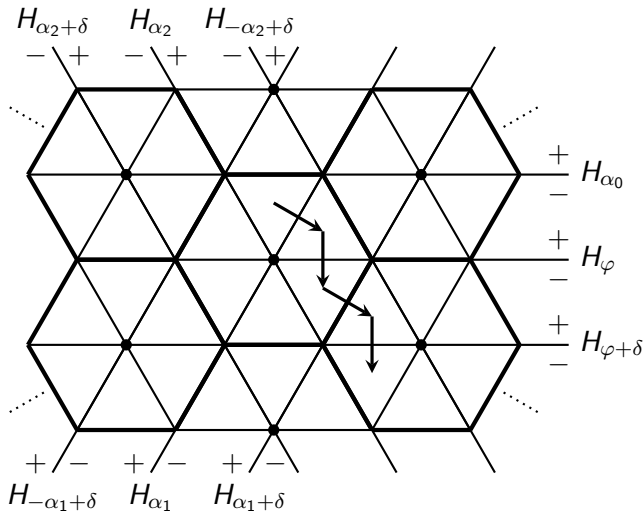
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We label each step of the walk with elements in a finite field \mathbb{F}_q .



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We label each step of the walk with elements in a finite field \mathbb{F}_q . It is a fact that

$$|w| \longleftrightarrow \left\{ \begin{array}{l} \text{walks to } w \text{ with} \\ \text{labels in } \mathbb{F}_q \end{array} \right\} .$$



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
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
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Turns steps of the form

$$H_{\pm\alpha_i + j\delta}$$


or

$$H_{\pm\alpha_i + j\delta}$$




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Turns steps of the form

$$H_{\pm\alpha_i+j\delta}$$

or

$$H_{\pm\alpha_i+j\delta}$$

into

$$H_{\pm\alpha_i+j\delta}$$

or

$$H_{\pm\alpha_i+j\delta}$$



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Turns steps of the form

$$\begin{array}{ccc}
 H_{\pm\alpha_i+j\delta} & & H_{\pm\alpha_i+j\delta} \\
 \begin{array}{c} + \quad - \\ | \\ \xrightarrow{c} \\ | \end{array} & \text{or} & \begin{array}{c} - \quad + \\ | \\ \xrightarrow{c} \\ | \end{array}
 \end{array}$$

into

$$\begin{array}{ccc}
 H_{\pm\alpha_i+j\delta} & & H_{\pm\alpha_i+j\delta} \\
 \begin{array}{c} + \quad - \\ | \\ \xleftarrow{c^{-1}} \\ | \end{array} & \text{or} & \begin{array}{c} - \quad + \\ | \\ \xleftarrow{c^{-1}} \\ | \end{array}
 \end{array}$$

If $c = 0$ we cannot fold and if $c \neq 0$ we must fold!



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Let $w = s_1 s_2 s_0 s_1$ with labels $(0, 0, c, 0)$.



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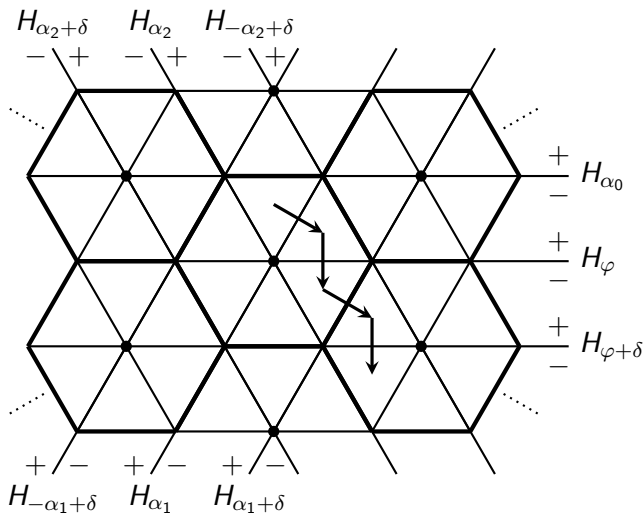
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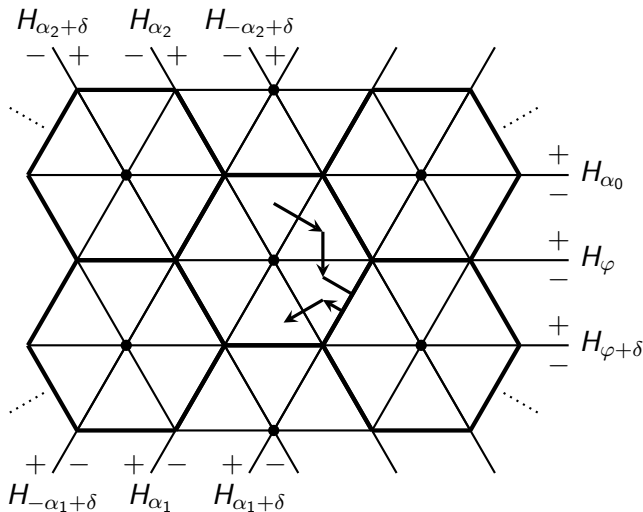
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Let $w = s_1 s_2 s_0 s_1$ with labels $(0, 0, c, 0)$. The folded walk is





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Two facts from Parkinson-Ram-Schwer:



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Two facts from Parkinson-Ram-Schwer:

$$\left\{ \begin{array}{l} \text{labeled walks to } w \\ \text{that positively fold to } v_1 \end{array} \right\} \longleftrightarrow U^- v_1 I \cap I w I.$$



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$$\left\{ \begin{array}{l} \text{labeled walks to } w \\ \text{that negatively fold to } v_2 \end{array} \right\} \longleftrightarrow U^+ v_2 I \cap I w I.$$



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From Beazley-Brubaker:

$$\left\{ \begin{array}{l} \text{labeled walks to } w \\ \text{that positively fold to } v_1 \\ \text{and negatively fold to } v_2 \end{array} \right\} \longleftrightarrow U^- v_1 I \cap I w I \cap U^+ v_2 I.$$



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$$\left\{ \begin{array}{l} \text{labeled walks to } w \\ \text{that negatively fold to } v_2 \end{array} \right\} \longleftrightarrow U^+ v_2 I \cap I w I.$$

From Beazley-Brubaker:

$$\left\{ \begin{array}{l} \text{labeled walks to } w \\ \text{that positively fold to } v_1 \\ \text{and negatively fold to } v_2 \end{array} \right\} \longleftrightarrow U^- v_1 I \cap I w I \cap U^+ v_2 I.$$

This last bijection makes our Whittaker functions extremely computable!



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We thank the NSF RTG grant supporting this work with grant no. DMS-1745638.

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